

Aharonov-Bohm protection of black hole's baryon/skyrmion hair

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The baryon/skyrmion correspondence implies that the baryon number is encoded into a topological surface integral. Under certain conditions that we clarify, this surface integral can be measured by an asymptotic observer in form of an Aharonov-Bohm phase-shift in an experiment in which the skyrmion passes through a loop of a probe string. In such a setup the baryon/skyrmion number must be respected by black holes, despite the fact that it produces no long-range classical field. If initially swallowed by a black hole, the baryon number must resurface in form of a classical skyrmion hair, after the black hole evaporates below a certain critical size. Needless to say, the respect of the baryon number by black holes is expected to have potentially-interesting astrophysical consequences.

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1 Introduction

Recently [1], we have argued that the skyrmion/baryon correspondence seriously challenges the standard conclusion of the “folk theorems” that the baryon number must be violated by black holes. We also argued that under the circumstances that will be elaborated in the present paper, this correspondence provides a “hidden” topological protection for the baryon number. The latter phenomenon is the main focus of the present paper. However, before presenting our analysis let us briefly recount the story.

One important fact for us is the existence of classical solutions of the Einstein equations describing black holes with classical skyrmion hair [2]. However, these solutions are only known in the domain of parameters in which the black hole horizon r_h is smaller than the characteristic skyrmion size L . As shown in [1], one can detect the classical skyrmion hair which is present in the regime $r_h \ll L$ through classical scattering experiments of waves scattered by the skyrmion black holes.

There are the two important aspects - classical and quantum - that the existence of skyrmion hair brings into the question of baryon/skyrmion number conservation in the presence of black holes.

First, as we have discussed in [1], although the very existence of a classically-observable skyrmion hair of a black hole with horizon size $r_h \ll L$ *a priori* does not guarantee the conservation of baryon/skyrmion number in the presence of black holes, nevertheless this conservation gets promoted into a self-consistent possibility: the skyrmion/baryon charge swallowed by a large black hole need not be lost, but instead can re-emerge in form of a black hole solution with classical skyrmion hair after the black hole - which initially swallowed a baryon - shrinks, due to Hawking evaporation, down to the size L .

The reason why we cannot make a stronger statement based on purely classical considerations is that in the parameter-domain in which the black hole horizon r_h largely exceeds the characteristic size of the skyrmion, L , no black hole solutions with classical skyrmion hair are known. Hence, it is impossible to classically monitor skyrmion/baryon charge when the black hole size exceeds L .

The quantum considerations bring a crucial new insight into the story. We have argued [1] that when the quantum effects are taken into the account, even in the domain $r_h \gg L$ the black hole can carry a full memory of the baryon/skyrmion number it had swallowed. This memory is kept in form of a topological boundary surface integral that - under certain conditions - can be measured by means of an Aharonov-Bohm phase-shift. Due to its topological nature this measurement is insensitive to a local state of the black hole-skyrmion system and allows to monitor the baryon/skyrmion charge of a black hole also in the regime $r_h \gg L$ in which the skyrmion hair is classically unobservable. Hence, in this situation, the baryon/skyrmion content must be revealed sooner or later.

If so, in what form this process should take place? Of course, we cannot exclude at this level of the discussion a possibility that the non-perturbative spec-

trum of gravity contains objects that are more compact than QCD-skyrmions and can carry the required skyrmion/baryon charge. In such a case, the locally-observable baryon/skyrmion charge can come out in form of these exotic creatures. However, we do not see the necessity for such a scenario.

From our point of view it is most natural to expect that the return of the “borrowed” baryon/skyrmion charge happens after the black hole shrinks to a size below L , where the known solutions with classical skyrmion hair do exist.

We must stress, however, that the end result of the black hole evolution is unimportant for the discussion of the present paper, since we focus exclusively on the topological and gauge constraints, which must be respected for all the cases.

2 Setup and ingredients

We shall first consider skyrmions in flat space, which is enough for capturing the essence of the topological protection. For simplicity, we restrict our analysis to the case of two quark flavors. For the metric we shall use the signature $(+, -, -, -)$.

The Skyrme Lagrangian in the case of two quark flavors is given by [3, 4, 5]

$$\mathcal{L}_{Skyrme} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_m, \quad (1)$$

with,

$$\mathcal{L}_2 = -\frac{F_\pi^2}{4} \text{Tr} (U^\dagger \partial_\mu U U^\dagger \partial^\mu U) \quad (2)$$

$$\mathcal{L}_4 = \frac{1}{32e^2} \text{Tr} ([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2) \quad (3)$$

$$\mathcal{L}_m = \frac{1}{2} m_\pi^2 F_\pi^2 (\text{Tr} U - 2) \quad (4)$$

where m_π is the pion mass and U is a $SU(2)$ matrix defined as,

$$U = e^{\frac{i}{F_\pi} \pi_a(x) \sigma_a}, \quad (5)$$

with $\pi_a(x)$ the pion fields and σ_a the Pauli matrices. F_π is the pion decay constant and e is the Skyrme coupling constant.

The above Lagrangian admits regular stable spherically-symmetric static configurations for which the pion field assumes a hedgehog form,

$$\frac{\pi_a}{F_\pi} = F(r) n_a, \quad (6)$$

where $n_a \equiv \frac{x_a}{r}$ is a unit vector in radial direction and $F(r)$ is a profile function with the boundary conditions,

$$F(0) = B\pi, \quad F(\infty) = 0. \quad (7)$$

Here B is an integer, which sets the topological charge of the soliton [4],

$$B = \int d^3x J_0, \quad (8)$$

where J_0 is the time-component of the skyrmion topological Chern-Simons current,

$$J_\mu = -\frac{\epsilon_{\mu\nu\alpha\beta}}{24\pi^2} \text{Tr} \left(U^{-1} \partial^\nu U U^{-1} \partial^\alpha U U^{-1} \partial^\beta U \right). \quad (9)$$

The explicit form of the solution-profile function is not important for our purposes and the discussions can be found in [4, 5].

The characteristic size and the mass of the skyrmion are given by

$$L = \frac{1}{eF_\pi} \text{ and } M_S = \frac{F_\pi}{e}, \quad (10)$$

respectively.

The next ingredient we shall need for our analysis is the skyrmion/baryon correspondence. Although originally the skyrmions were introduced by Skyrme [3] as description for baryons, the correspondence is best understood in $SU(N_C)$ QCD with large number of colors after work by Witten [6]. We shall therefore work in the regime $N_C \gg 1$.

In this case there exists a one-to-one correspondence between the Nöther baryonic current in theory of quarks q_f ($f \equiv$ flavor index),

$$J_\mu = \frac{1}{N_C} \sum_f \bar{q}_f \gamma_\mu q_f, \quad (11)$$

and the topological Chern-Simons current (9) in the chiral effective theory of pions.

Hence, in this sense a baryon carries a topological skyrmion charge. The key point now is to notice [1] that this charge can be measured at infinity, despite the fact that neither a baryon nor the skyrmion produces a locally-observable classical field at $r \rightarrow \infty$. Nevertheless, there exists a possibility of detecting the skyrmion charge in Aharonov-Bohm type experiments.

In order to rewrite the baryon charge as a surface integral, let us first notice that the skyrmion topological current (9) represents a Hodge-dual of the exterior derivative of a two-form which takes - when evaluated on the hedgehog ansatz (6) with the boundary conditions (7) - the following form

$$S_{\mu\nu} \equiv -\frac{1}{4\pi^2} \left(F(r) - \frac{1}{2} \sin(2F(r)) - \pi \right) \partial_{[\mu} \cos(\theta) \partial_{\nu]} \phi. \quad (12)$$

Here r, θ and ϕ are the usual spherical coordinates and $F(r)$ is the profile function, with the boundary conditions $F(0) = \pi$ and $F(\infty) = 0$. For simplicity, we have chosen the $B = 1$ case of (7). The choice of the constant π in (12) is uniquely dictated by the requirement that the two-form $S_{\mu\nu}$ is well-defined

everywhere. This is how the information about the topological charge is encoded into this two-form. The value of the charge is insensitive to a particular form of the function $F(r)$, but only to its boundary values.

The topological charge (8) is then given by an integral over a boundary surface enclosing the skyrmion, which can be taken to be a two-sphere of infinite (or sufficiently-large) radius r , and with embedding coordinates X^μ ,

$$B = \int_{S_2} dX^\mu \wedge dX^\nu S_{\mu\nu} = 1. \quad (13)$$

This gives the correct representation of the skyrmion charge (8) in form of the boundary surface integral.

3 Measuring the baryon number by Aharonov-Bohm type experiments

Despite the fact that there is no locally-observable classical field asymptotically, the topological baryonic charge is nevertheless observable via an Aharonov-Bohm type measurement. Such a measurement can be performed by coupling the two-form $S_{\mu\nu}$ to a string, via the usual coupling between the string world-sheet element and an antisymmetric two-form,

$$\mathcal{S}_{string} = g \int dX^\mu \wedge dX^\nu S_{\mu\nu}, \quad (14)$$

where X^μ are string embedding coordinates and g is a coupling constant. The precise nature of the string is unimportant for our present analysis. For example, its role can be played by a cosmic string, a fundamental string, or any other string-like object, e.g., a flux-tube of some gauge field. Later we shall give an example of a particular microscopic resolution of the probe string in form of a cosmic string. However, at the moment we shall work at the level of effective theory and treat the probe string as fundamental.

Any physical process in which the string world-volume encloses the skyrmion, will result into an Aharonov-Bohm effect with the phase shift given by,

$$\Delta\Phi = 2\pi g, \quad (15)$$

and will be observable provided g is not an integer.

For example, we can consider an interference of the two string loops in the process in which the skyrmion passes through one of them (see Fig. 1 for illustration). Alternatively, we can consider a vacuum process in which a virtual string loop gets created on a north pole of an imaginary two-sphere surrounding the skyrmion, encloses the sphere and collapses at the south pole (see Fig. 2 for illustration).

Thus, in such a setup an asymptotic observer can monitor the baryon charge in form of the Aharonov-Bohm skyrmion hair at infinity. Now it is clear that black holes must respect this charge. Indeed, consider a Gedankenexperiment

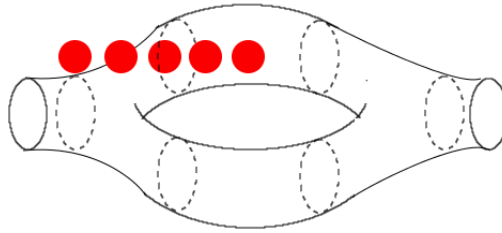


Figure 1: Interference of two strings with a skyrmion (illustrated by the red circle) passing through one of them

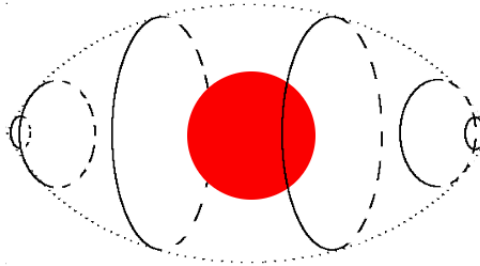


Figure 2: A virtual string loop surrounding a skyrmion (illustrated by the red circle)

in which a skyrmion whose topological charge has been measured at infinity ends up inside a black hole of horizon size $r_h \gg L$. Although there is no known solution with classical skyrmion hair outside of such a black hole, nevertheless the information about the skyrmion charge is carried by the Aharonov-Bohm phase of the conserved charge (13), and therefore, cannot be lost.

Hence, after evaporation such a black hole is forced to reveal its baryon/skyrmion content in some form. Of course, it is most natural to assume - as it was discussed in [1] - that the reemergence of the baryon charge will take place in form of the known solutions with classical skyrmion hair, once the black hole shrinks below the size L . However, irrespectively whether nature chooses this possibility or some yet unknown way, the baryonic charge must be respected due to its topological protection by the Aharonov-Bohm phase.

4 The secret gauge structure

The possibility of detecting black hole quantum numbers via an Aharonov-Bohm measurement has been observed previously for the following cases: for two-form gauge fields both massless [7] and massive [8], for discrete gauge symmetries [10] and for a massive spin-2 field [9]. The common feature in all these examples is the existence of an elementary gauge field, which assumes a pure gauge form asymptotically. This looks very different from the present case, since the Skyrme Lagrangian (1) does not contain any elementary gauge

degree of freedom¹.

We observe that despite the fact that the Skyrme Lagrangian has no propagating gauge degree of freedom, the two-form $S_{\mu\nu}$ effectively assumes the role analogous to the one played by a gauge potential in an ordinary Aharonov-Bohm effect.

In order to see that the analogy is deeper than what may naively seem, notice that the Skyrme topological current (9) is invariant under the gauge shift of the two-form $S_{\mu\nu}$ by an exterior derivative of an arbitrary one-form $\zeta_\mu(x)$,

$$S_{\mu\nu} \rightarrow S_{\mu\nu} + \partial_\mu \zeta_\nu(x) - \partial_\nu \zeta_\mu(x). \quad (16)$$

Notice, that the above gauge-redundancy is itself redundant since $S_{\mu\nu}$ experiences the same shift under a family of ζ_ν -parameters related to each other by an $U(1)_\zeta$ gauge transformation

$$\zeta_\nu \rightarrow \zeta_\nu + \partial_\nu \alpha(x), \quad (17)$$

where $\alpha(x)$ is an arbitrary non-singular function.

If we for a moment think about ζ_μ as of a vector-potential of this gauge $U(1)_\zeta$ -symmetry, then from the point of view of this $U(1)_\zeta$ -theory the skyrme configuration of the $S_{\mu\nu}$ (12) for $r \rightarrow \infty$ coincides with the abelian field-strength of Dirac's magnetic monopole: $S_{\mu\nu} = \partial_{[\mu} \zeta_{\nu]}$, where the one-form ζ_ν can be chosen as,

$$\zeta_\nu = \frac{1}{4\pi} (\cos(\theta) - 1) \partial_\nu \phi. \quad (18)$$

In this description the skyrmion charge is effectively identified with a magnetic charge of $U(1)_\zeta$ magnetic monopole. This way of visualizing things is not creating any new conserved quantity, but it helps to understand the skyrmion charge in the language of the hidden gauge structure.

Of course, as we know very well, in the case of a $U(1)_\zeta$ magnetic monopole in order to deliver a non-zero magnetic charge the parameter ζ_ϕ must become singular somewhere. For example, in the gauge (18) this singularity manifests itself in form of a Dirac string oriented along the $z < 0$ semi-axes. If ζ_ν were a Maxwellian gauge field, the Dirac string would be rendered unobservable by imposing the Dirac quantization condition on elementary electric charges that source ζ_ν .

In our case the Dirac string is automatically unobservable since ζ_ν is not a gauge degree of freedom, but a gauge redundancy parameter for $S_{\mu\nu}$. Due to this there exist no elementary particles that are electrically charged under

¹Another difference of the present example from the cases of [7], [8] and [9] is that in the latter cases no non-singular flat space solutions exist, and such a hair cannot exist without a black hole, due to the need of hiding the singularity behind the black hole horizon. Correspondingly, it is not immediately clear what should be the end result of evaporation of a black hole that carries such a hair. In contrast, in the skyrmion case, the non-singular solutions do exist in the flat space in form of skyrmions and represent a fully consistent candidate for the end-product of the black hole evaporation. This feature is related to the fact that the object $S_{\mu\nu}$ that plays the role of the Aharonov-Bohm hair-carrying gauge field in the skyrmion case is not an elementary field. It can assume a topologically non-trivial asymptotically pure-gauge form without causing a singularity in the skyrmion solution (see below).

the “field” ζ_ν . Correspondingly, the Dirac string for the $U(1)_\zeta$ monopole is by-default unphysical.

The above is the key to understanding of why - despite the fact that the skyrmion charge can be visualized as the $U(1)_\zeta$ -monopole charge - the skyrmion solution is regular everywhere. The singularity in the $U(1)_\zeta$ -monopole configuration can be attributed to the parameter that parameterizes the redundancy (17) of the redundancy (16). This is most transparent in the Wu-Yang formulation [11] in which in upper ($0 < \theta < \frac{\pi}{2}$) and lower ($\frac{\pi}{2} < \theta < \pi$) hemispheres the parameter ζ can be represented as follows

$$\zeta_\nu^{(U)} = \frac{1}{4\pi} (\cos(\theta) - 1) \partial_\nu \phi, \quad \zeta_\nu^{(L)} = \frac{1}{4\pi} (\cos(\theta) + 1) \partial_\nu \phi, \quad (19)$$

with $\zeta_\phi^{(U)}$ and $\zeta_\phi^{(L)}$ at the equator differing by a single-valued gauge transformation (17) with $\alpha = \phi$,

$$\left(\zeta_\nu^{(L)} - \zeta_\nu^{(U)} \right)_{\theta=\frac{\pi}{2}} = \frac{1}{2\pi} \partial_\nu \phi. \quad (20)$$

That is, the redundancy parameter ζ_ν is patched by using its own gauge-redundancy (17).

Thus, the $U(1)_\zeta$ -magnetic monopole configuration that $S_{\mu\nu}$ assumes for $r \rightarrow \infty$ corresponds - when viewed from the point of view of the gauge redundancy (16) - to a *locally-pure-gauge* form. $S_{\mu\nu}$ assumes this form only asymptotically and smoothly departs from it at the origin ($r \rightarrow 0$) without encountering a singularity.

To summarize, the Aharonov-Bohm phase-shift (15) effectively measures the magnetic charge of an “embedded” $U(1)_\zeta$ -Dirac monopole created by the vector gauge-parameter ζ_ν . Of course, in reality there is no classically-observable magnetic field, since ζ_ν is not a physical degree of freedom, but a parameter of the gauge redundancy (16) of the skyrmion charge. Thus, from the point of view of the two-form gauge symmetry (16) the skyrmion configuration is asymptotically locally *pure-gauge* and the Aharonov-Bohm phase-shift takes place due to the non-trivial topology, just as in case of an ordinary Aharonov-Bohm phenomenon.

5 Manufacturing a probe string

Here we shall give an example of manufacturing a probe string that is charged under the skyrmion two-form $S_{\mu\nu}$.

Following [9], let us show that the role of such a string can be played by a Nielsen-Olesen cosmic string of Abelian Higgs model.

The Lagrangian has a standard form,

$$\mathcal{L} = |D_\mu H|^2 - \lambda^2 (|H|^2 - v^2)^2 - F_{\mu\nu} F^{\mu\nu}. \quad (21)$$

Here H is the complex scalar, $D_\mu \equiv \partial_\mu - iqA_\mu$ is the covariant derivative and $F_{\mu\nu} \equiv \partial_{[\mu} A_{\nu]}$ is the field strength. q is the gauge coupling. The system is

invariant under the gauge symmetry,

$$H \rightarrow e^{i\omega(x)} H, \quad A_\nu \rightarrow A_\nu + \frac{1}{q} \partial_\nu \omega(x), \quad (22)$$

where ω is a gauge-transformation parameter.

The non-zero vacuum expectation value (VEV) of the scalar, $\langle |H| \rangle = v$, generates a mass gap and the resulting spectrum of the theory consists of a gauge boson of mass qv and a scalar of mass λv . The phase of H becomes a longitudinal polarization of the massive “photon” A_μ .

It is well known [12] that this theory admits the string-like soliton solutions, the so-called Nielsen-Olesen vortex lines which can be viewed as quantum field theoretic versions of Abrikosov vortices in superconductors. These strings represent the tubes of quantized magnetic flux with the elementary unit of magnetic flux given by $flux = \oint A = \frac{2\pi}{q}$, where the line integral is taken along a closed path around the string. The thickness of the magnetic flux-tube is of the order of the Compton wavelength of the gauge field, $\sim (qv)^{-1}$.

In order for the Nielsen-Olesen string to act as a source for the skyrmion two-form, we need to introduce the following coupling,

$$c S_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}, \quad (23)$$

where c is a parameter. At the level of an effective theory, the above coupling is fully legitimate and is compatible with all the symmetries of the problem. For example, it is invariant under the gauge shift of the skyrmion two-form (16).

If we now perform the above-described Aharonov-Bohm-type measurement of the skyrmion charge by using the loop of the cosmic string, the resulting phase-shift will be equal to

$$\Delta\Phi = 2\pi \frac{c}{q}. \quad (24)$$

This is not surprising, since for the string loops larger than their thickness the coupling (23) reduces to an effective coupling (14) with $g = \frac{c}{q}$,

$$c S_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta} \rightarrow \frac{c}{q} \int dX^\mu \wedge dX^\nu S_{\mu\nu}. \quad (25)$$

Thus, the existence of strings with fractional two-form charge g is determined by the ratio c/q .

The reason of why the coupling (23) between the skyrmion two-form $S_{\mu\nu}$ and the abelian field strength results into the observability of the skyrmion charge by boundary measurement can be understood in the following way. The coupling (23) implies that the skyrmion current sources A_μ . In this way, the skyrmion becomes effectively *electrically* charged under the massive A_μ and the Aharonov-Bohm effect in which the string loop encloses the skyrmion can be equivalently understood as the “ordinary” Aharonov-Bohm effect in which an object (i.e., the skyrmion) which is electrically charged with respect to A_μ goes through the loop of a solenoid (i.e., the cosmic string) that carries the magnetic flux of A_μ .

For the rational fractional values of c/q , we can interpret the situation in the following way. By introducing the coupling (23) the baryon/skyrmion acquires a fractional charge under the gauge $U(1)$ -symmetry. The VEV of the Higgs is invariant under a discrete subgroup Z_N , where N is the minimal integer number for which Nc/q is an integer. Thus, the skyrmion acquires a discrete gauge hair ala Krauss and Wilczek [10]. For irrational values of c/q , the subgroup is formally Z_∞ and the skyrmion acquires an infinite discrete hair.

An alternative useful interpretation of why in the presence of the coupling (23) the skyrmion acquires a fractional charge under the Higgsed $U(1)$ -symmetry (22), is in terms of Witten's effect [13]. As explained above, the asymptotic configuration of $S_{\mu\nu}$ is given by the field strength of a magnetic monopole of ζ_ν . Thus, asymptotically the coupling (23) reduces to a dual coupling between the field-strengths of the A_μ and ζ_μ vectors,

$$c S_{\mu\nu} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta} \rightarrow c F_{\mu\nu}^{(\zeta)} F_{\alpha\beta} \epsilon^{\mu\nu\alpha\beta}, \quad (26)$$

where $F_{\mu\nu}^{(\zeta)} \equiv \partial_{[\mu} \zeta_{\nu]}$. As a result, the $U(1)_\zeta$ -monopole acquires an electric charge c/q under the gauge symmetry (22) through Witten-type effect.

Notice, for our purposes of external monitoring of the skyrmion hair, the coupling c in (23) can be taken arbitrarily small.

6 Concluding remarks

We conclude with some remarks in order to point out possible generalizations of our analysis, to emphasize the relevance of the effective-theory framework we are working in, to point out the importance of the large- N_C approximation and to mention possible further implications of our results.

First, for simplicity we restricted our analysis to the case of a single skyrmion with topological charge $B = 1$. One can generalize our arguments both for the case with more $B = 1$ skyrmions as well as for the case of a single skyrmion with $B > 1$. For a general value B of the skyrmion charge the two-form (12) takes the form

$$S_{\mu\nu} \equiv -\frac{1}{4\pi^2} \left(F(r) - \frac{1}{2} \sin(2F(r)) - B\pi \right) \partial_{[\mu} \cos(\theta) \partial_{\nu]} \phi. \quad (27)$$

The Aharonov-Bohm phase shift is then $\Delta\Phi = 2\pi gB$. Thus, if g is a rational number, one can always choose the value of B for which $gB = n$, with n an integer, and the resulting phase shift $\Delta\Phi = 2\pi n$ is unobservable. This can never be achieved if g is irrational. In our effective field theory treatment, since the string is a spectator and g can be taken to be arbitrary, the phase shift can be made observable for any value of the skyrmion charge.

Second, we want to emphasize that both our arguments in favor of baryon/skyrmion number conservation by semi-classical black holes [1] and the discussion about

the measurement of the baryon/skyrmion number of a black hole by the Aharonov-Bohm type experiments which we described in this paper are based on a *chiral low-energy effective theory of pions*. Our arguments do neither imply that baryon number is necessarily conserved at the level of a fundamental theory operating at higher energy scales, nor do they imply that such a high-energy theory necessarily allows to write down a coupling of the form (14) with g non-integer.

In fact, there are well-known examples in which operators violating baryon number are generated by high-energy physics (for example this is the case in grand-unified theories like $SU(5)$).

If the effective chiral theory with skyrmions is at higher energies embedded in such a baryon-number violating theory, by consistency, the same theory must provide a super-selection rule that forbids the coupling of the form (14) with g non-integer. In such a case no baryon number can be measured in the Aharonov-Bohm type experiments described above, because either the coupling (14) will not exist or the generated phase-shift would always be an integer multiple of 2π .²

Third, we work with a large number of colors $N_C \gg 1$. In this regime the baryon/skyrmion correspondence is best understood. For smaller values of N_C it was expected that the correspondence between baryons and skyrmions still holds, but that skyrmions which correspond to the baryons require a quantum-corrected description [6]. These quantum corrections are expected to appear as finite- N_C corrections. In pure $SU(N_C)$ -QCD the baryon number is an anomaly-free symmetry and must be conserved in full quantum theory. Thus, as long as the baryon/skyrmion correspondence holds, the finite- N_C corrections (either $1/N_C$ or $\exp(-N_C)$ type) are not expected to destabilize the skyrmion. In pure QCD the topological stability of the skyrmion could be jeopardized only by the effects that abolish the baryon/skyrmion correspondence. The instability scale will then be determined by the strength of these effects.

Further, we would like to comment that the skyrmion/baryon hair discussed in the present work is different from the quantum baryonic hair of a black hole suggested earlier in [16] in the context of a specific microscopic theory. The two ideas are fully consistent and may very well turn out to be complementary, but at no point in the present work we have made any assumptions about the quantum properties of a black hole. The beauty of the skyrmion/baryon hair is that it does not rely on any particular microscopic picture of a black hole

²In cases when the high energy theory violates baryon number, this violation also penetrates in the low energy theory of pions. This is the case for example [14] in a system with $SU(5)$ -magnetic monopole, which catalyses baryon number violation [15]. As shown in [14], in such a case, the violation of baryon number by the high energy theory translates in the low energy theory of pions as non-existence of a conserved gauge invariant topological current which would be well-defined everywhere. Therefore, in such a case, it is not possible to write down a two-form of the type (12) which would be well-defined everywhere and thus no corresponding conserved charge can be monitored at infinity.

and employes solely the power of topology and gauge redundancy, which must be respected by any consistent microscopic theory.

Finally, it would be important to study further the possible astrophysical implications of the topological protection of baryon number against black holes.

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